

# Probability and Random Processes

## ECS 315

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### 6.3 Bernoulli Trials



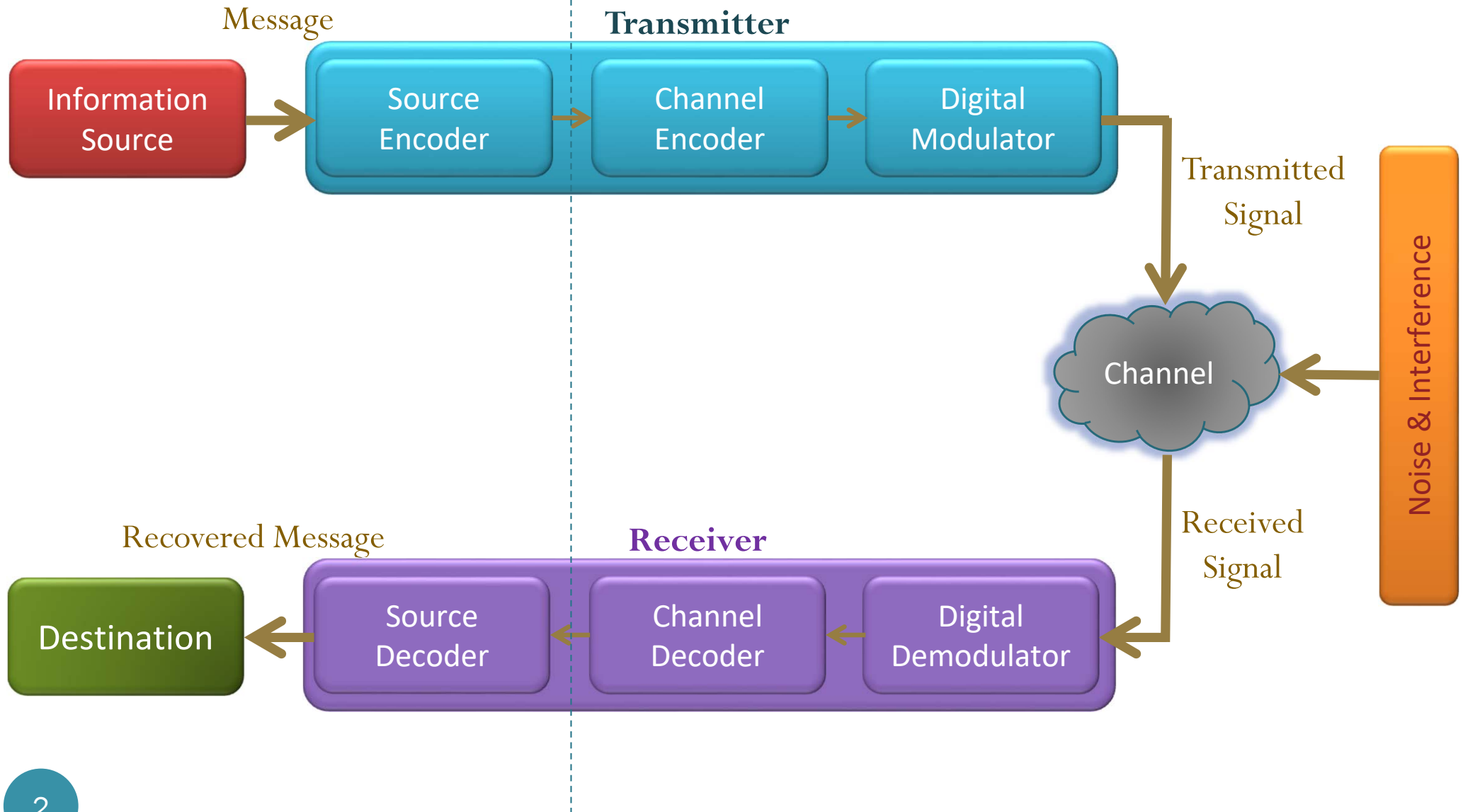
#### Office Hours:

BKD, 6th floor of Sirindhralai building

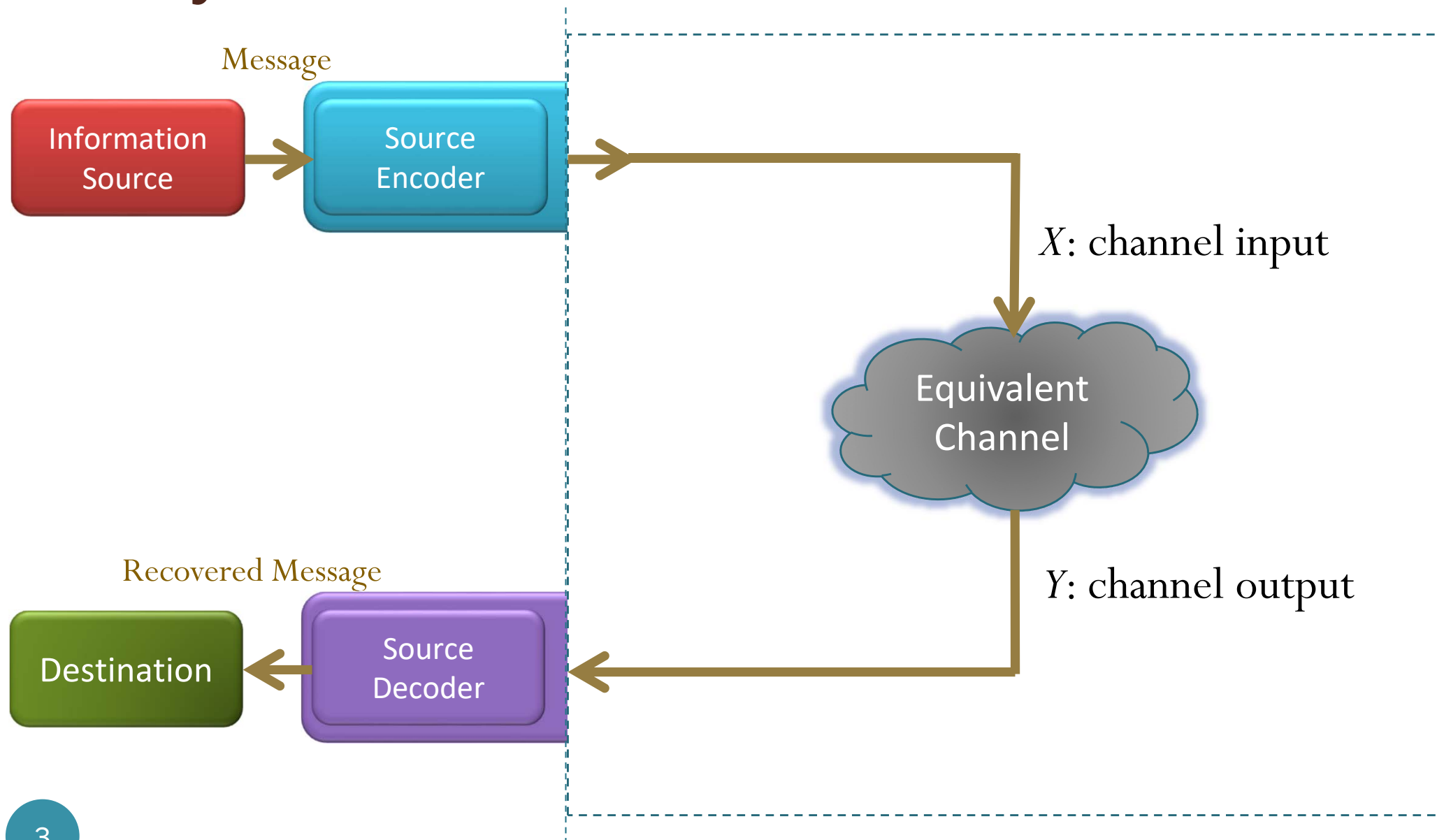
Wednesday 14:30-15:30

Friday 14:30-15:30

# Elements of digital commu. sys.



# System considered



# The ASCII Coded Character Set

				6	0	0	0	0	1	1	1	1							
<i>Bit</i>				5	0	0	1	1	0	0	1	1							
<i>Number</i>				4	0	1	0	1	0	1	0	1							
				1st	0	1	2	3	4	5	6	7							
3	2	1	0	Hex															
				2nd															
0	0	0	0	0	0	NUL	16	DLE	32	SP	48	0	64	@	80	P	96	112	p
0	0	0	1	1	1	SOH		DC1		!		1		A		Q		a	q
0	0	1	0	2	2	STX		DC2		"		2		B		R		b	r
0	0	1	1	3	3	ETX		DC3		#		3		C		S		c	s
0	1	0	0	4	4	EOT		DC4		\$		4		D		T		d	t
0	1	0	1	5	5	ENQ		NAK		%		5		E		U		e	u
0	1	1	0	6	6	ACK		SYN		&		6		F		V		f	v
0	1	1	1	7	7	BEL		ETB		'		7		G		W		g	w
1	0	0	0	8	8	BS		CAN		(		8		H		X		h	x
1	0	0	1	9	9	HT		EM		)		9		I		Y		i	y
1	0	1	0	A	A	LF		SUB		*		:		J		Z		j	z
1	0	1	1	B	B	VT		ESC		+		;		K		[		k	{
1	1	0	0	C	C	FF		FS		,		<		L		\		l	
1	1	0	1	D	D	CR		GS		-		=		M		]		m	}
1	1	1	0	E	E	SO		RS		.		>		N		^		n	~
1	1	1	1	F	F	SI		US		/		?		O		_		o	DEL

# Example: ASCII Encoder

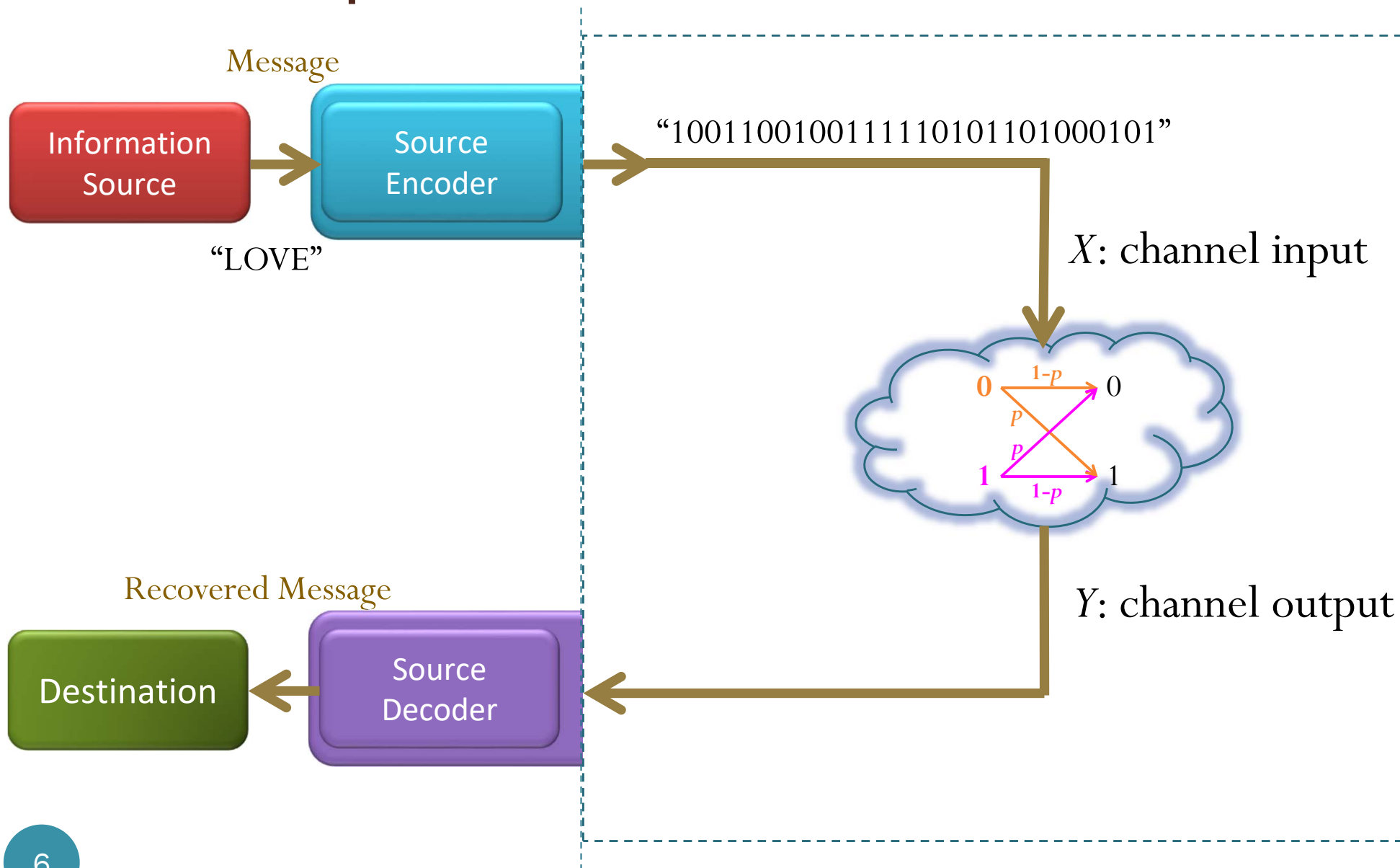
Character	Codeword
:	
E	1000101
:	
L	1001100
:	
O	1001111
:	
V	1010110
:	

MATLAB:

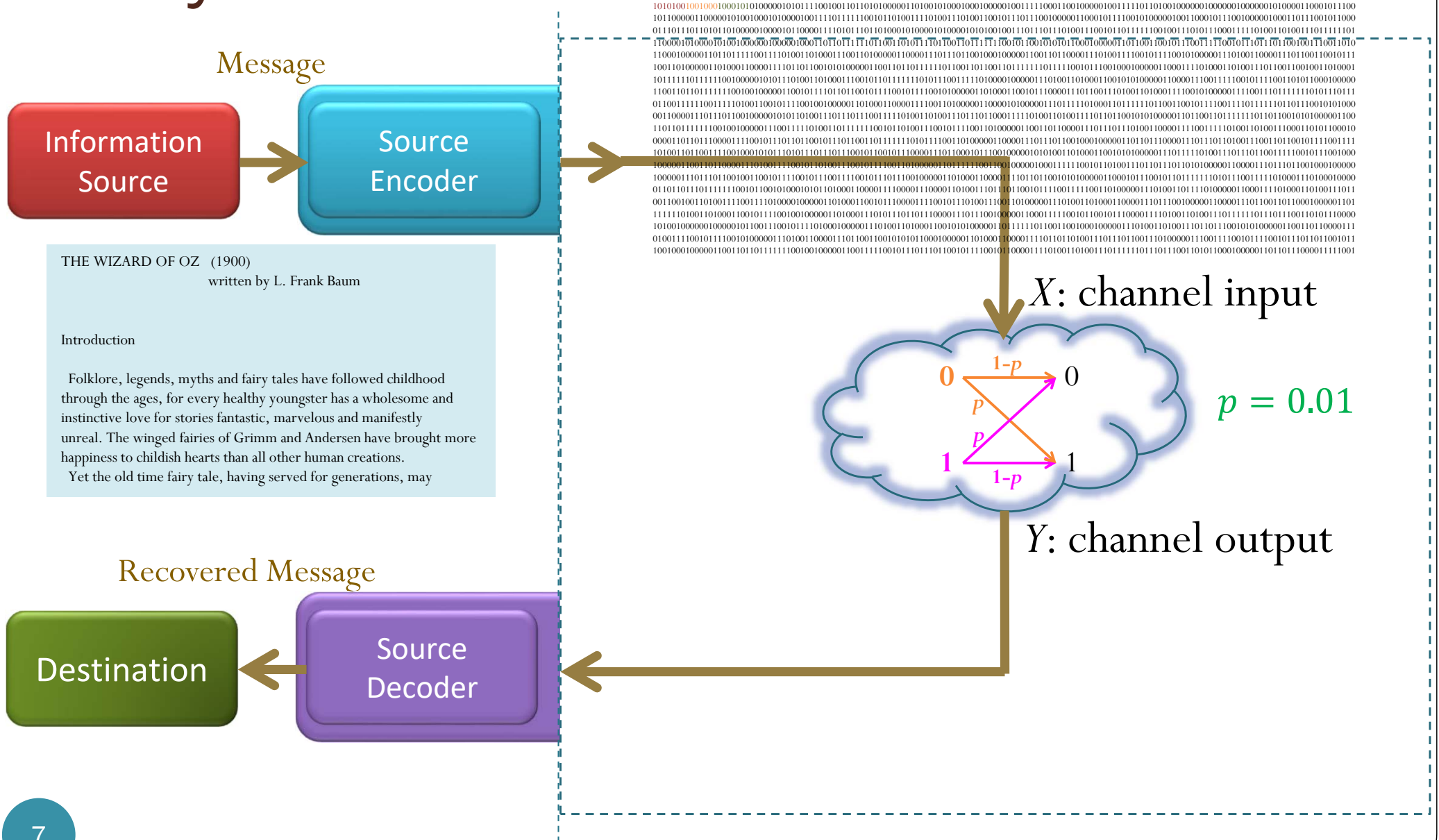
```
>> M = 'LOVE';  
>> X = dec2bin(M, 7);  
>> X = reshape(X', 1, numel(X))  
X =  
1001100100111110101101000101
```



# Example: ASCII Encoder and BSC



# System considered







# Results

THE WIZARD OF OZ (1900)  
written by L. Frank Baum

## Introduction

Folklore, legends, myths and fairy tales have followed childhood through the ages, for every healthy youngster has a wholesome and instinctive love for stories fantastic, marvelous and manifestly unreal. The winged fairies of Grimm and Andersen have brought more happiness to childish hearts than all other human creations. Yet the old time fairy tale, having served for generations, may

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## Introduction

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- The whole book which is saved in the file “OZ.txt” has 207760 characters (symbols).
- The ASCII encoded string has  $207760 \times 7 = 1454320$  bits.
- The channel corrupts 14545 bits.
- This corresponds to 14108 erroneous characters (symbols).

# Results

```
>> ErrorProbabilityoverBSC
biterror =
  14545
BER =
  0.010001237691842
theoretical_BER =
  0.0100000000000000
symerror =
  14108
SER =
  0.067905275317674
theoretical_SER =
  0.067934652093010
```


$$\frac{14545}{1454320} \approx 0.01 \quad \leftarrow$$


$$\frac{14108}{207760} \approx 0.0679 \quad \leftarrow$$

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# Results

BER  $p = 0.01$

$$\frac{14545}{1454320} \approx 0.01$$


$$\frac{14108}{207760} \approx 0.0679$$


$$\text{SER} = 1 - (1 - p)^7$$

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- This corresponds to 14108 erroneous characters (symbols).

A character (symbol) is successfully recovered if and only if none of its bits are corrupted.

# Crossover probability and readability

Whef th% &i2sv nkvdI"On(txE"serm-s< HaRtY Qo |p%R\$anlthe Phi\$)qop8gb'rYtoNe (puclirhed  
in23/ee c uNpr9es aZ Harby!PovDdZ qnd0THA!Uorojev's Qpof'), pegsL iT is0aqazenP'Tiet` {nle  
sau\*!fICQ~t eve.t`xA# raken pOqb%%)D }Hm`wizprdYjv"wOrnd--a~%W%Jv s' tury  
2maskABdd\$`eden(tl| LuxGxec`nOlike c)gzq of ktTiu!f5mm"cackG@ Ud(to"vhhQ a~aNd alt  
tn0vid veRckn of HaRvq\$Xntter#isxohk { regea,ed@&saduadLy u(2otGh"tau griEs."AfTex0T`g  
mntr DUCt ry kh`ter,\$thd(fomN0j`apv ngrwarTt-0c t,me"1xortly bEemsL |ar2q Pnfter'3  
aMen-n5i@Fipth\$`q, aoh It i3d1t piac0pmhnP d\*if Zas mafibin"je#k7poUndpb%dins tk`be qe6e!lgd.

$p = 0.10$  → SER  $\approx 0.52$

When phe fir v okval ov"th% serie3, @`rry0Pntter efdxtxe Thil soph%rs Stone0(p}blisjed!in  
{ooe c un |pye { agav0y Potter aj` (the sorcerer"s S4o |e)< opdns- mt"Is!apParEnt 4hat somu  
siwnidiga.v evant iAs take."plhge in(uhe w)zard)ng wo { |d--An event so very Rumar {ablel  
eteN0Dhe %ugcles\$n t)ce signs of\$At. Tje!&ul |!backep/und Dk thkw`event ajt(to vhd per {On  
of8Ikxry P\_Pter is oN,y rereAeud gredualli 4hroufh5ie qeriesn Af|ir the )~trofUctkry!ciapter,\$tle  
r%ook lE`ps for erd8to!a d)hg 3Hostly redobd HARRY(Potter/r elaventl(birpl%ay,))nd(iD i3 1t tlis  
hohlt vhat iis\$magical bac+gropnd bedans to bg rEve!ied/

$p = 0.05$  → SER  $\approx 0.30$

# Crossover probability and readability

When the first novel of the series, *Harry Potter and the Philosopher's Stone* (published in some countries as *Harry Potter and the Sorcerer's Stone*), opens, it is apparent that a significant event has taken place in the wizarding world--an event so very remarkable, even the Muggle notices signs of it. The full background to this event and the person of Harry Potter is only revealed gradually through the series. After the introductory chapter, the book leaps forward to a time shortly before Harry Potter's eleventh birthday and it is at this point that his magical background begins to be revealed.

$$p = 0.03 \rightarrow \text{SER} \approx 0.19$$

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$$p = 0.02 \rightarrow \text{SER} \approx 0.13$$

# Crossover probability and readability

Human may be able to correct some (or even all) of these errors.

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$$p = 0.01 \rightarrow \text{SER} \approx 0.07$$

# Crossover probability and readability

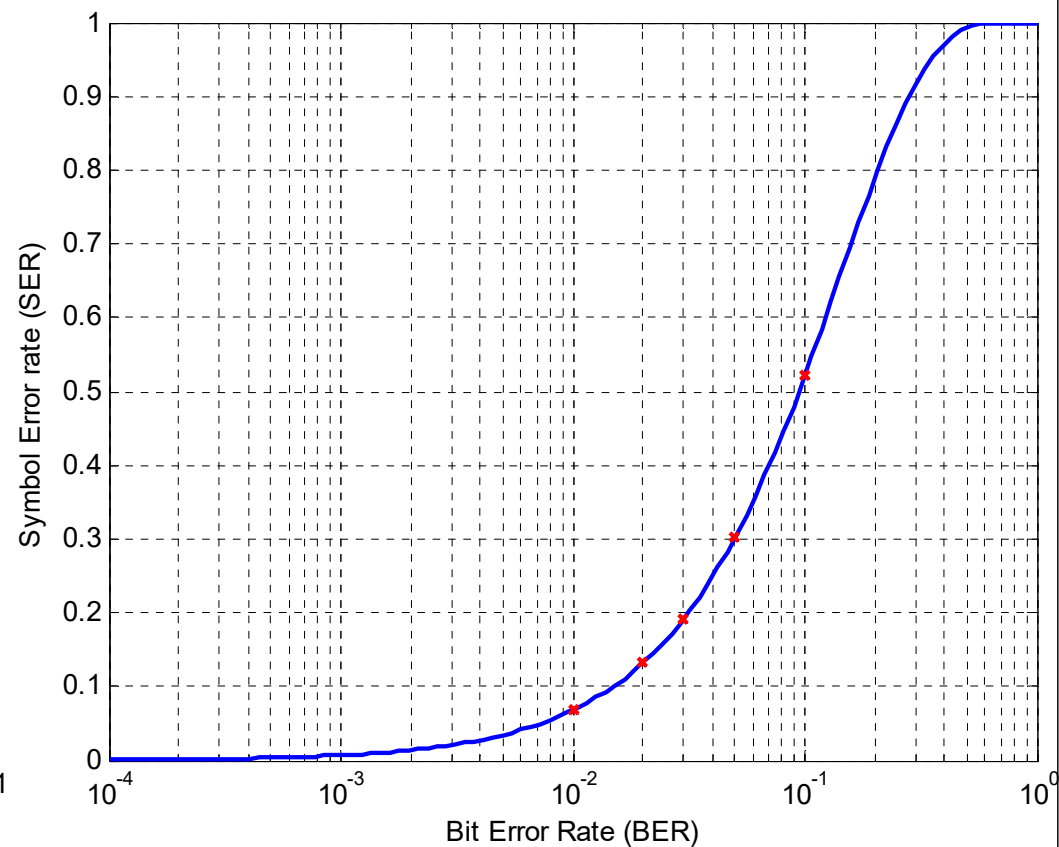
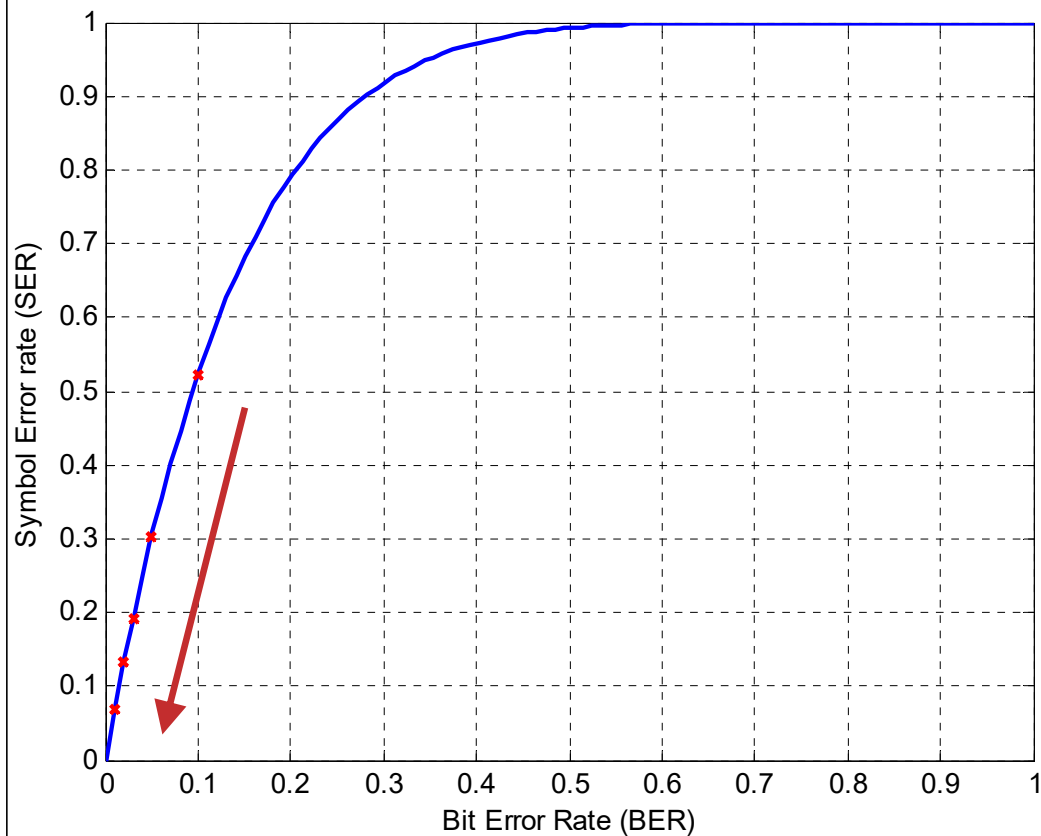
When the first novel of the series, Harry Potter and the Philosopher's Stone (published in some countries as Harry Potter and the Sorcerer's Stone), opens, it is apparent that some significant event has taken place in the wizarding world--an event so very remarkable, even the Muggles notice signs of it. The full background to this event and to the person of Harry Potter is only revealed gradually through the series. After the introductory chapter, the book leaps forward to a time shortly before Harry Potter's eleventh birthday, and it is at this point that his magical background begins to be revealed.

Original

When the first novel of the series, Harry Pottez and the Philosopher's Stone (p5blished in some countries as Harry Potter cnd the Sorcerep's Stone), opens, it i3 apparent that soMe cignifacant event!haS taken0place in the wi~arding 7orld--ao event so `very!bemark!blu, even the Mufgles nodice signs" of it. The fuld background to this event and to the person of Harry P/tTer is only revealed gradually through th series. After the introfuctory chapter, the boo+ leaps forward to a time shortly before Harpy Potteb7s eleventh`birthday, and )t is at this poi~t that his -agikal bac{ground begins to be revealed.

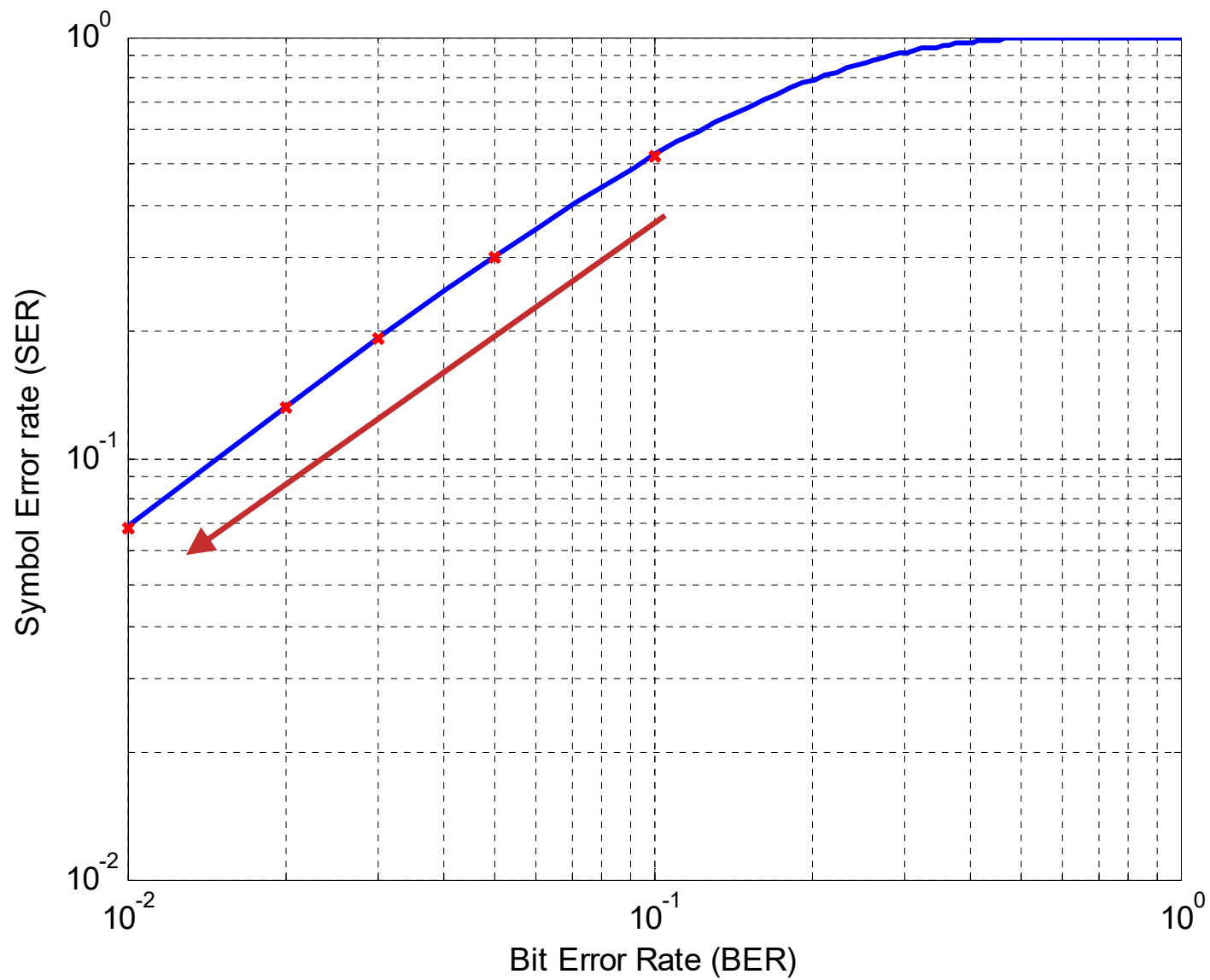
$p = 0.01$  → SER  $\approx 0.07$

# BER vs. SER

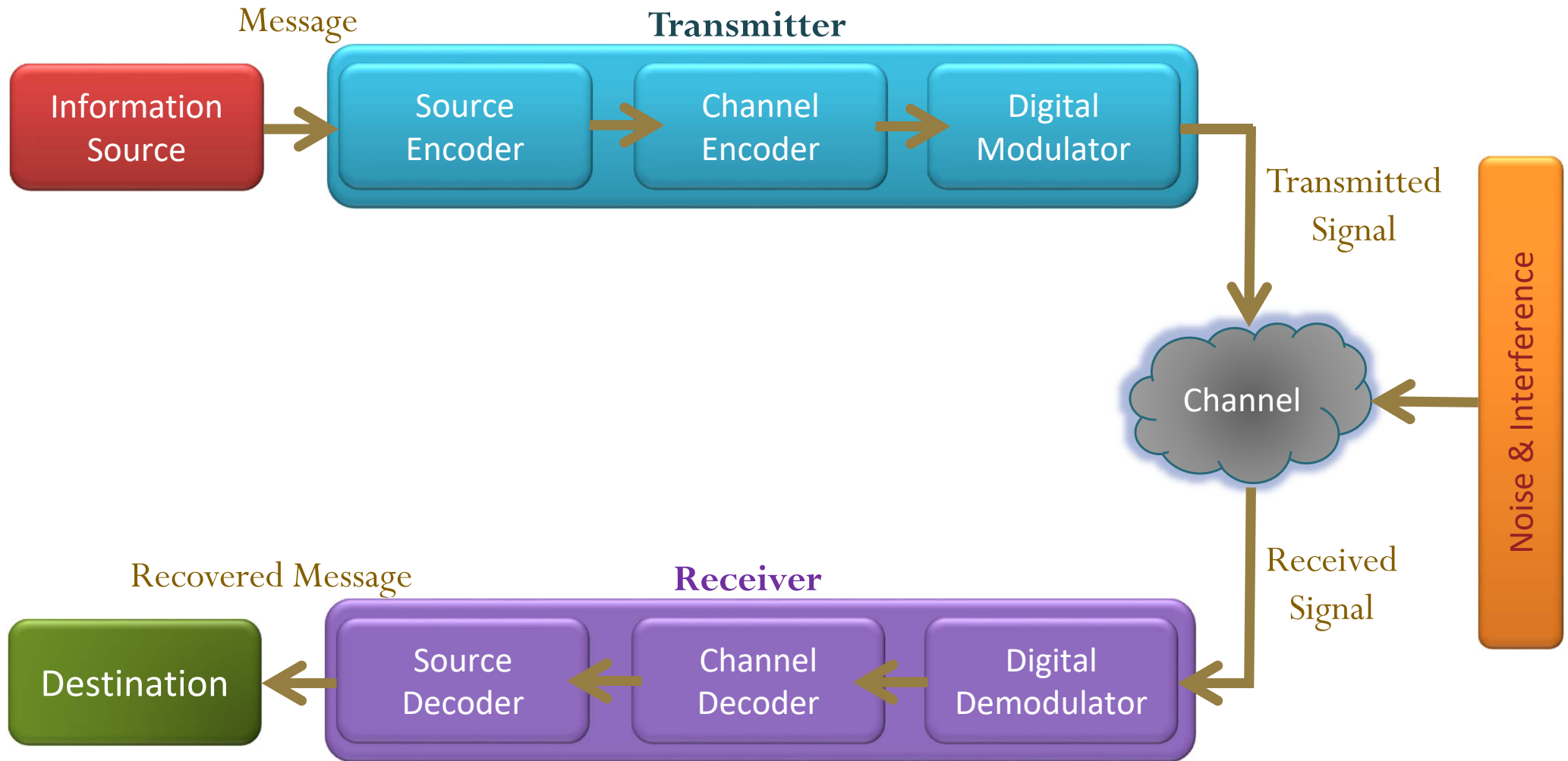




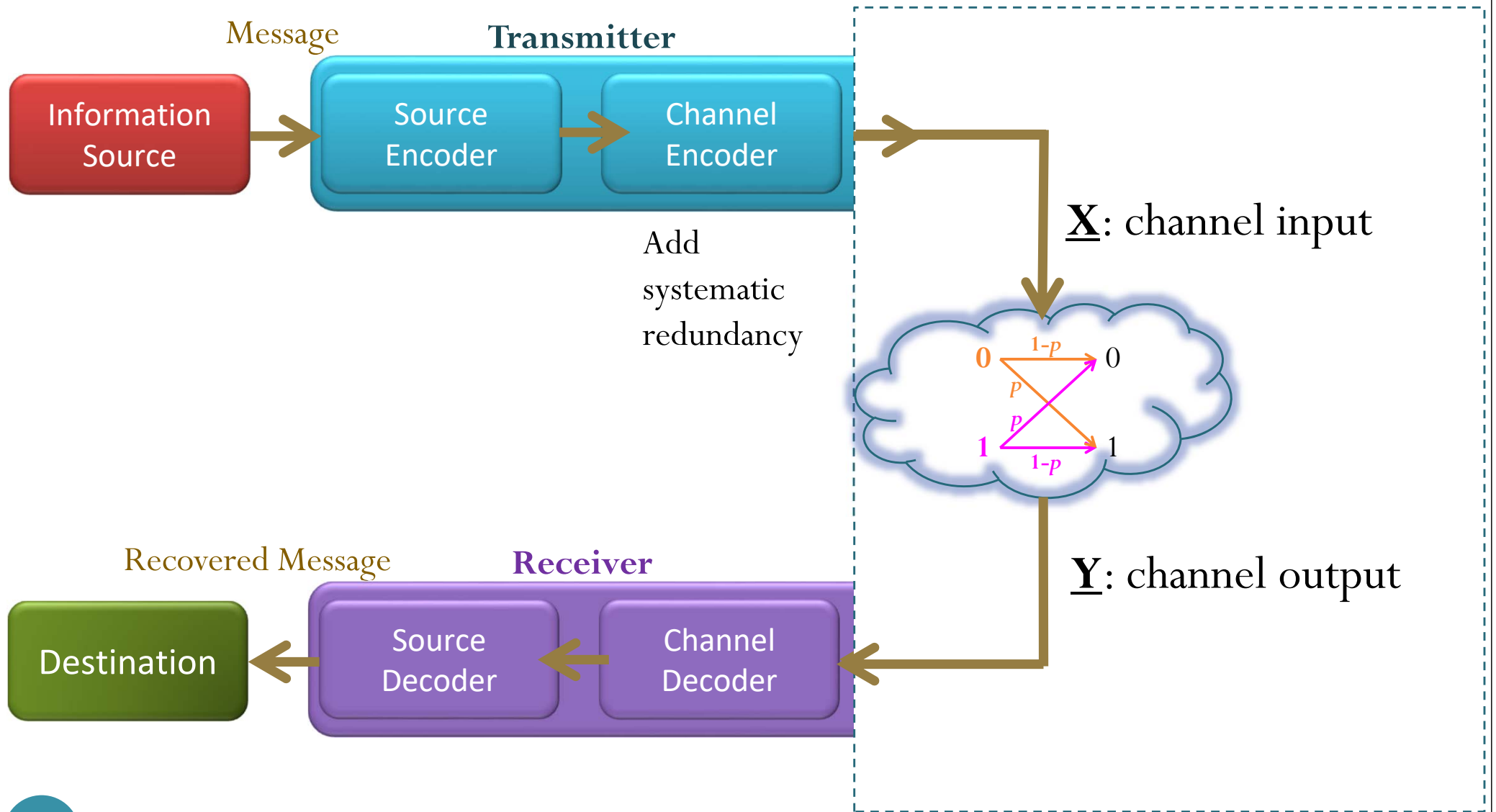
# BER vs. SER



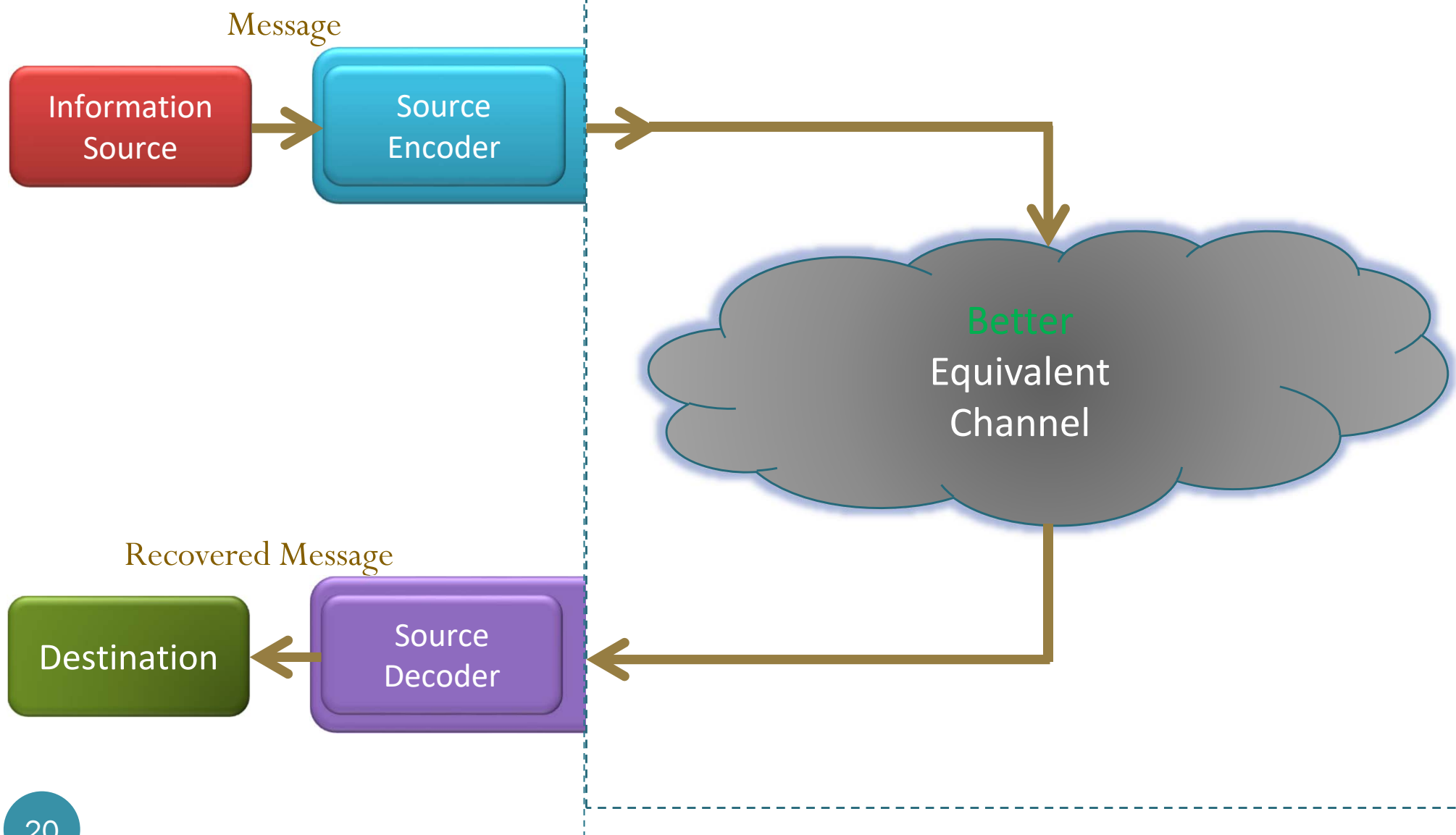
# Elements of digital commu. sys.



# Channel Encoder and Decoder

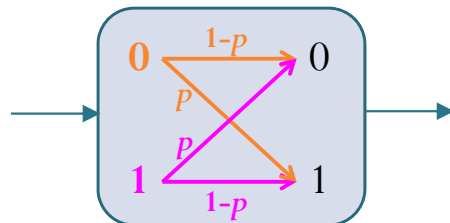


# Better Equivalent Channel



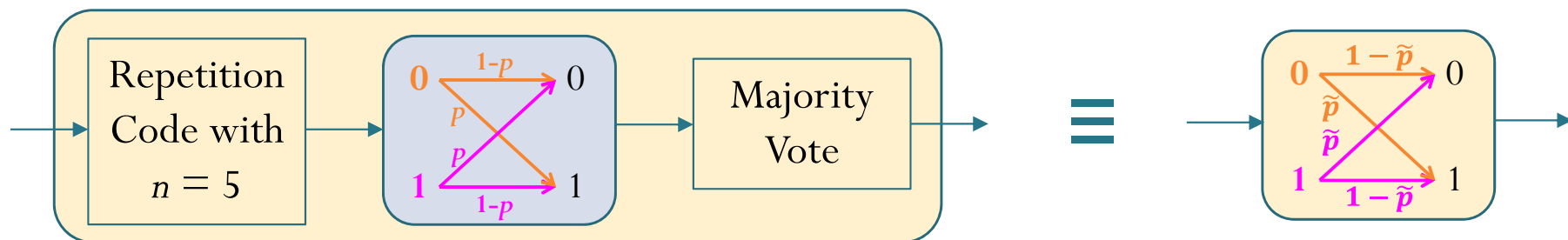
# Example: Repetition Code

- Original Channel:



- BSC with crossover probability  $p = 0.01$

- New (and Better) Equivalent Channel:



- Use repetition code with  $n = 5$  at the transmitter
- Use majority vote at the receiver
- New BSC with  $\tilde{p} = \binom{5}{3}p^3(1-p)^2 + \binom{5}{4}p^4(1-p)^1 + \binom{5}{5}p^5(1-p)^0 \approx 10^{-5}$